Math 1050 Mortgage Project, Fall 2011

Names Brittney Evans | Chase Call **Due date:** 11/17/2011

In this project we will examine a home loan or mortgage. Assume that you have found a home for sale and have agreed to a purchase price of **$210,000.**

Down Payment: Assume that you are going to make a 10% down payment on the house. Determine the amount of your down payment and the balance to finance.

Down Payment $21,000 Mortgage Amount $189,000

Part I: 30 year Mortgage

Monthly Payment: Calculate the monthly payment for a 30 year loan (rounding up to the nearest cent) by using the following formula . Show your work. [PMT is the monthly loan payment, P is the mortgage amount, r is the annual percent rate for the loan *in decimal*, and Y is the number of years to pay off the loan. For the 30 year loan use an annual interest rate of 3.875%.

$$PMT = \frac{P\left(\frac{r}{12}\right)}{1-\left(1+\frac{r}{12}\right)^{-12Y}}$$

$$\frac{\$189{,}000\left(\frac{.03875}{12}\right)}{1-\left(1+\frac{.03875}{12}\right)^{-12(30)=-360}} = \frac{610.3125}{.6867} = 888.75$$

Monthly Payment for a 30 year mortgage = $888.75

Note that this monthly payment covers only the interest and the principal on the loan. It **does not cover** any insurance or taxes on the property.

Amortization Schedule: In order to summarize all the information regarding the amortization of a loan, construct a schedule that keeps track of the payment number, the principal paid, the interest, and the unpaid balance. A spreadsheet program is an excellent tool to develop an amortization schedule. We can use a free amortization spreadsheet on the web.
The web address is: http://www.bretwhissel.net/amortization/amortize.html. Enter the **amount of the loan**, i.e. the selling price minus the down payment, the **interest rate**, and the appropriate **number of years.** Check the box to show the schedule. If you are making extra payments towards the principal, include it in the monthly payment and leave the number of payments box blank.

Amortization Schedule monthly payment for a 30 year mortgage = $898.75
(Note: if this is more than 2 or 3 cents different from your calculation, check your numbers!)

Total interest paid over 30 years = $130,950

Total amount repaid = $319,950

Notice that the amount of the payment that goes towards the principal and the amount that goes towards the interest are not constant. What do you observe about each of these values?
The principle is lower than the interest at first, but as the principle goes up the interest goes down.

Find the number of the first payment when more of the payment goes toward principal than interest.
147 payment - principle $446.01 - interest $442.94

As already mentioned, these payments are for principal and interest only. You will also have monthly payments for home insurance and property taxes. In addition, it is helpful to have money left over for those little luxuries like electricity, running water, and food. As a wise home owner, you decide that your monthly principal and interest payment should not exceed 28% of your monthly take-home pay. What minimum monthly take-home pay should you have in order to meet this goal? Show your work for making this calculation.

$$(100)\ \frac{898.75}{28\%} = \frac{x}{100\%}\ (28)$$

$$\frac{89875}{28} = \frac{28x}{28}$$

Minimum monthly take home pay = $3174.10.

It is also important to note that your net or take-home pay (after taxes) is less than your gross pay (before taxes). Assuming that your net pay is 73% of your gross pay, what minimum gross annual salary will you need to make to have the monthly net salary stated above? Show your work for making this calculation.

$$\frac{3174.10}{73} = \frac{x}{100}$$

$$\frac{(12)x}{12} = 4,348.082192\ (12)$$

$$x = 52176.99$$

Minimum gross annual salary = $52,176.99

Part II: Selling the House

Let's suppose that after living in the house for 10 years, you want to sell. The economy experiences ups and downs, but in general the value of real estate increases over time. To calculate the value of an investment such as real estate, we use continuously compounded interest.

Find the value of the home 10 years after purchase assuming a continuous interest rate of 3.6%. Use the full purchase price as the principal. Show your work.

$a = Pe^{rt}$

$a = 210{,}000\, e^{.036(10)}$

$a = \$300{,}999.18$

Assuming that you can sell the house for this amount, use the following information to calculate your gains or losses:

Selling price of your house $\$300{,}999.18$

Original down payment $\$21{,}000$

Mortgage paid over the ten years $\$106{,}650$

The principal balance on your loan after ten years $\$148{,}269.21$

$\$275{,}919.21$

Do you gain or lose money over the 10 years? How much? Show your amounts and summarize your results:

you gain: $\$25{,}079.97$

Total exp after 10 years is 10 years: $\$275{,}919.21$

Sell the house for $\$300{,}999.18$ and gain $\$25{,}079.97$

Part III: 15 year Mortgage

Using the same purchase price and down payment, we will investigate a 15 year mortgage.

Monthly Payment: Calculate the monthly payment for a 15 year loan (rounding up to the nearest cent) by using the following formula . Show your work. [PMT is the monthly loan payment, P is the mortgage amount, r is the annual percent rate for the loan *in decimal*, and Y is the number of years to pay off the loan. For the 15 year loan use an annual interest rate of 3.125%.

$$PMT = \frac{P\left(\frac{r}{12}\right)}{1-\left(1+\frac{r}{12}\right)^{-12Y}} \qquad \frac{189{,}000\left(\frac{.03125}{12}\right)}{1-\left(1+\frac{.03125}{12}\right)^{-12(15)}} = \frac{492.1875}{.373} = 1316.59$$

Monthly Payment for a 15 year mortgage = $1,316.59

Use the amortization spreadsheet on the web again, this time entering the interest rate and number of payments for a 15 year loan.

Amortization Schedule monthly payment for a 15 year mortgage = $1,316.59
(Note: if this is more than 2 or 3 cents different from your calculation, check your numbers!)

Total interest paid over 15 years = $47,986.20

Total amount repaid = $236,986.20

Find the number of the first payment when more of the payment goes toward principal than interest.

It's always more for principal.

Suppose you paid an additional $100 towards the principal each month.

How long would it take to pay off the loan with this additional payment?

164 payments , 13.6 years.

What is the total amount of interest paid over the life of the loan?

$43,465.08

Compare this total amount repaid to the total amount repaid without any extra payments. How much more or less would you spend if you made the extra principal payments?

you would pay $4,521.12 LESS if you paid $100 more each month.

October 19, 2011

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Transportation Costs (or Trucker's Dilemma)

Math 1050 College Algebra Project

A truck driving 260 miles over a flat interstate at a constant rate of 50 miles per hour gets 7 miles to the gallon. Fuel costs $3.50 per gallon. For each mile per hour increase in speed, the truck loses a tenth of a mile per gallon in its mileage. Drivers get $27.50 per hour in wages and fixed costs for running the truck amount to $11.33 per hour. What constant speed (between 50 mph and the speed limit of 65 mph) should the truck drive to minimize the total cost of the trip?

Part I: To solve a problem like this, it is a good idea to start with calculating an actual example. We will begin by finding the total cost if the truck is driven at 50 miles per hour. The cost is made up of two components: the fuel cost and the time cost. Be sure to include the correct units with each value below.

A. Let's start out by finding how long the trip will take.

$\frac{260 \text{ miles}}{50 \text{ MPH}} = 5.2$

The length of time required for the trip is 5.2 hours. *(Do not round.)*

B. Now, with this time known, how much will it cost to pay the driver and run the truck?

$= (5.2 \text{ hours})(\$27.50 + \$11.33)$

$= 5.2 \text{ hours}(\$38.83)$

$= 201.916$

The amount needed to pay the driver and run the truck is $201.92. *(Round to nearest cent.)*

C. Next determine, at 7 miles per gallon for 260 miles, how much fuel will be required.

$\frac{260 \text{ miles}}{7 \text{ MPG}}$

The amount of fuel required is $\frac{260}{7}$ gallons. *(Do not round. Leave as a fraction.)*

D. With the amount of fuel known, how much will the fuel cost to make the 260 miles?

$\frac{260}{7} \text{ gallons}(\$3.50) = 130$

The cost of the fuel is $130.00. *(Round to nearest cent.)*

E. Finally we can find the TOTAL cost.

$130.00 fuel cost

$201.92 driver/truck cost

$331.92

The total cost for the trip is $331.92. *(Round to nearest cent.)*

Part II: The preceding process should have illuminated the basic procedure we will use to find the total cost of a trip. Next we will find the total cost if the truck is driven at 65 miles per hour. As in Part I, include the correct units with each value.

A. Let's find how long the trip will take.

$\frac{260 \text{ miles}}{65 \text{ MPH}} = 4$

The length of time required for the trip is 4 hours. *(Do not round.)*

B. Now, with this time known, how much will it cost to pay the driver and run the truck?

$= (4 \text{ hours})(\$27.50 + \$11.33)$

$= 4 \text{ hours} (\$38.83)$

$= 155.32$

The amount needed to pay the driver and run the truck is $155.32. *(Round to nearest cent.)*

C. Next, to begin determining the fuel cost, we need to find the mileage (miles per gallon of fuel) when the truck is travelling at 65 miles per hour.

$= 7 \text{ MPG} - \left(\frac{1}{10}(65 \text{ MPH} - 50 \text{ MPH})\right) \text{MPG}$

$= 7 \text{ MPG} - \left(\frac{1}{10}\right)(15)$

$= 7 \text{ MPG} - 1.5$

$= 5.5 \text{ MPG}$

The mileage at 65 miles per hour is 5.5 MPG. *(Do not round.)*

D. With the fuel mileage known, how much fuel will be needed for the 260 miles?

$\frac{260 \text{ miles}}{5.5 \text{ MPG}}$

The amount of fuel required is $\frac{260}{5.5}$ gallons. *(Do not round. Leave as a fraction.)*

E. With the amount of fuel known, how much will the fuel cost to make the 260 miles?

$\frac{260 \text{ miles}}{5.5 \text{ MPG}} \left(\frac{\$3.50}{1 \text{ gallon}}\right) = 165.4545$

The cost of the fuel is $165.45. *(Round to nearest cent.)*

F. Finally we can find the TOTAL cost.

$165.45 fuel cost

$155.32 driver/truck cost

$320.77

The total cost for the trip is $320.77. *(Round to nearest cent.)*

Part III. We should now have a good process for determining the total cost of a trip for any rate of speed greater than or equal to 50 miles per hour. Next is to create a Total Cost function using X as the unknown rate in miles per hour. Simplify your answers and remember to include units. *As you work through each step, test your answers by plugging in 50 mph and then 65 mph and comparing with results from parts I and II.*

A. Let's find how long the trip will take.

$\frac{260 \text{ Miles}}{X \text{ MPH}} = \frac{260}{X} \text{ hours}$

The length of time required for the trip is $\frac{260}{X}$ hours.

B. Now with this time known, how much will it cost to pay the driver and run the truck?

$= \frac{260}{X} \text{ hours } (\$27.50 + \$11.33)$

$= \frac{260}{X} (\$38.83)$

The amount of money needed to pay the driver and run the truck is $\frac{10095.8}{X}$.

C. Next, to begin determining the fuel cost, we need to find the mileage (miles per gallon of fuel) when the truck is travelling at X miles per hour.

$f(x) = 7 - (x - 50)(\frac{1}{10})$

$= 7 - (\frac{1}{10}x - 5)$

$= 12 - \frac{1}{10}x$

The mileage at X miles per hour is $f(x) = -\frac{1}{10}x + 12$.

D. With the fuel mileage known, how much fuel will be needed for the 260 miles?

$\frac{260 \text{ Miles}}{(-.1x+12) \text{ MPG}} = \frac{260}{-.1x+12}$

The amount of fuel required is $\frac{260}{-.1x+12}$ gallons.

E. With the amount of fuel known, how much will the fuel cost to make the 260 miles?

$\frac{260}{-.1x+12} \left(\frac{\$3.50}{\text{gallon}}\right) = \frac{910}{-.1x+12}$

The cost of the fuel is $\frac{910}{-.1x+12}$.

F. Now we can find the TOTAL cost function. Express your function as C(X) =

$\$\left(\frac{10095.8}{X}\right) + \left(\frac{910}{-.1x+12}\right) = \$ \text{ total}$

TOTAL Trip Cost Function is $\left(\frac{10095.8}{X}\right) + \left(\frac{910}{-.1x+12}\right)$

G. The last thing we should do is verify that this is the correct function by evaluating it at 50 mph and 65 mph to see if we get the same values we have previously computed.

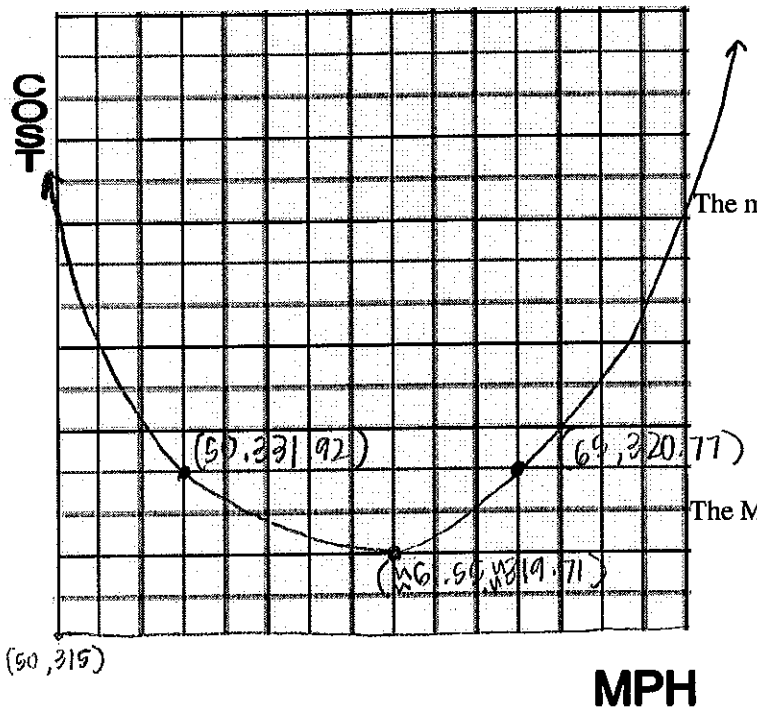
$$F(50) = \left(\frac{10095.8}{50}\right) + \left(\frac{910}{-.1(50)+12}\right) = 331.916$$

F(50) = $331.92

$$F(65) = \left(\frac{10095.8}{65}\right) + \left(\frac{910}{-.1(65)+12}\right) = 320.7745$$

F(65) = $320.77

Part IV. Assuming the function is modeling correctly, you need to calculate the minimum cost. Graph the Cost Function and find its minimum point. Sketch your graph here: Have the lower left point represent (50,315). You may use a graphing utility to help you find the minimum point.



The minimum COST for the trip is ≈ $319.71

The MPH that minimizes COST is ≈ 61.55.

Reflective Writing: How did this project change the way you think about real-world math applications? Write at least one paragraph stating what ideas changed and why. Next, discuss how the math skills that you applied in this project will impact other classes you will take in your school career? Point to specific parts of the project and your own process in completing it that might have applications for other classes.